Supporting learning resource — Part-part-whole reasoning — Addition and subtraction

Why is part-part-whole reasoning important?
Part-part-whole thinking refers to how numbers can be split into parts. It allows students to see the relationship between a number and its component parts. As a result, students generalise the connections between addition and subtraction.

Part-part-whole reasoning is a critical underpinning concept required for flexible and efficient computation. It also assists students’ attempts to visualise, interpret and solve word problems.

How does part-part-whole reasoning develop?
Part-part-whole thinking typically develops before children commence formal schooling. Young children (3-4 year olds) can often subitise small collections (quantities of 3 or 4), e.g. they know how many in a small collection without having to count. In fact, the ability to subitise can develop before children have developed efficient counting skills.

Young children who are able to subitise a collection (e.g. 4 apples) might describe the collection as ‘two and two and that’s four’.

They recognise the ‘parts’ of four as 2 and 2 and use this to quantify the ‘whole’ collection, without the need for counting.

At school, in their explorations with numbers, students are encouraged to represent and think of numbers in different ways, e.g. eight as 5 and 3, or 4 and 4, or 3 and 3 and 2. Structured materials are commonly used to develop this early, flexible part-part-whole thinking, including the use of ten frames and counters.

Part-part-whole relationships can then be represented in simple number sentences and diagrams, e.g.

\[
8 = 4 + 4
\]
As students encounter larger numbers, they will recognise the significance of partitioning numbers into place value parts. These part-part-whole relationships can also be represented with a variety of structured materials, models and diagrams.

Standard place value partitioning reflects the individual values of each digit in a number, e.g.

245 as

Non-standard place value partitioning shows flexible ways to break numbers up, e.g.

327 as

Part-part-whole reasoning and computation
Students first see the value of part-part-whole thinking in recalling addition number facts.

For example, a student is presented with the addition fact: $7 + 5$.
Using mental partitioning, the student has at least three efficient options.

- Think of 5 as 3 and 2. Add 7 and 3 (compatible numbers). Add the 2 to make 12.
- Think of 7 as 5 and 2. Add 5 and 5 (makes 10). Add the 2 to make 12.
- Think of 7 as 6 and 1. Add the 1 to the 5. Now add 6 and 6 (double fact).

Students use part-part-whole thinking to describe addition and subtraction situations. They learn to represent situations by acting them out, using materials or visualising the situation (creating a mental image). They can then interpret situations in part-part-whole terms.

For example, consider this situation:

- Seven children are inside the classroom.
- Twelve children have gone out to have lunch.
Students can determine the value of the parts in this situation (seven and twelve). The value of the whole is unknown. Students can calculate the value of the whole (nineteen).

Part-part-whole understanding and language allows students to: interpret problems, describe them as addition or subtraction and select appropriate strategies to solve them, e.g.

- 7 students are inside. 12 are at lunch. How many children are in the class? (addition situation, solved by adding: 7 + 12)
- There are 19 children in the class. 12 went out to lunch. How many are left? (subtraction situation, solved by subtracting: 19 – 12 or adding: 12 + □ = 19)
- 19 children are in the class. 7 were left in the room. How many went to lunch? (subtraction situation, solved by subtracting: 19 – 7 or adding: 7 + □ = 19)

From experiences in describing, interpreting and solving problems using part-part-whole reasoning, students develop the following key understandings:

- Addition and subtraction are related.
- Addition is commutative, but subtraction is not.
- The same situation can often be represented by addition and subtraction.

Some students find it difficult to solve addition and subtraction word problems because they are unsure of the appropriate operation to use. Part-part-whole reasoning can assist these students.

Typically, addition and subtraction word problems require students to identify a missing value. Students who are able to identify the missing (unknown) value as a ‘missing part’ or ‘missing whole’ will often be more successful in choosing an appropriate operation.

Example 1:

**24 sheep were in the paddock. 54 sheep were on the truck.**

**How many sheep were there in all?**

Students can represent this situation with materials, drawings or diagrams to show that both parts of this situation are known. The whole is unknown. This can be represented in a part-part-whole model as:

```
  ?
  24 54
```

Example 2:

**78 sheep were to be transported on the truck. 54 had already been loaded.**

**How many more sheep were to go on the truck?**
Students can represent this situation to show that *the whole and one part are known*. **One part is unknown.** This can be represented in a part-part-whole model as:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>78</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>54</td>
</tr>
</tbody>
</table>

Students will derive two simple, but highly valuable rules for solving word problems:

1. If both parts are known and the whole is unknown, use **addition**.
2. If the whole and one part are known, use **subtraction**.

**Note:** While subtraction is likely to be the most efficient operation to solve problems involving a ‘missing part’, students may choose to use open addition, e.g. $54 + \square = 78$. 